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A GEOMETRICAL REPRESENTATION FOR THE HIGH FREQUENCY DIELECTRIC TENSOR OF A TEMPERATE PLASMA

by Sidney Brooks

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Cleveland, Ohio

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SUMMARY

A new geometrical representation for the high-frequency dielectric tensor of a temperate plasma is given. It is simple in form and readily shows the effect of varying the plasma parameters.

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INTRODUCTION

In the monograph by Allis, Buchsbaum, and Bers (ref. 1), a geometrical construction introduced by Deschamps and Weeks (ref. 2) is used to represent the complex dielectric tensor of a temperate plasma at high frequencies. It is possible to modify this construction slightly in order to obtain a representation which is mathematically and diagrammatically simpler.

TENSOR COMPONENTS

The diagonal components of the dielectric tensor, $K_{||}$, K_{ℓ}, and K_r, are given in terms of the reduced plasma frequency α_-, the reduced electron cyclotron frequency β_-, and the reduced collision frequency γ_- by the equations

$$K_{||} = 1 - \frac{\alpha_-^2}{1 - j\gamma_-} \tag{1}$$

$$K_{\ell} = 1 - \frac{\alpha_-^2}{1 + \beta_- - j\gamma_-} \tag{2}$$

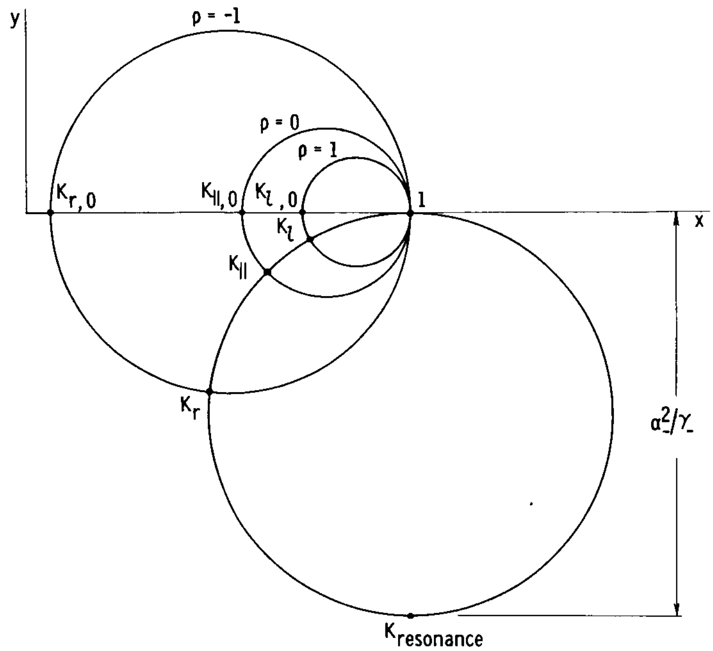

Figure 1. - Dielectric tensor for case in which wave frequency is greater than cyclotron frequency.

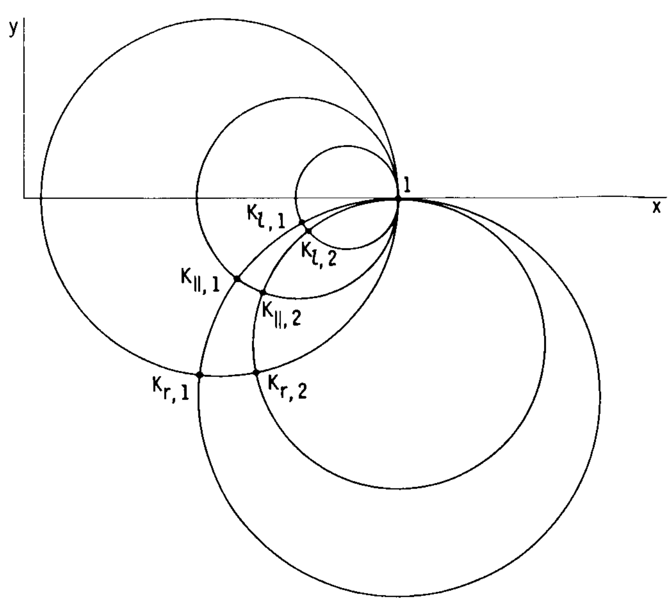

Figure 2. - Effect of changing collision frequency; coefficients $K_{r,2}$, $K_{||,2}$, and $K_{l,2}$ correspond to higher collision frequency.

$$K_r = 1 - \frac{\alpha_-^2}{1 - \beta_- - j\gamma_-} \tag{3}$$

These three equations can be written in the form

$$K = 1 - \frac{\alpha_-^2}{1 + \rho\beta_- - j\gamma_-} \tag{4}$$

where $\rho = 0$, 1, and -1 for $K_{||}$, K_ℓ, and K_r, respectively. (All symbols are defined in the appendix.) Since K is a complex number, it can be written in terms of its real and imaginary parts:

$$K = X + jY \tag{5}$$

By simple algebra, it follows that

$$X = 1 - \frac{\alpha_-^2(1 + \rho\beta_-)}{(1 + \rho\beta_-)^2 + \gamma_-^2} \tag{6}$$

$$Y = \frac{-\alpha_-^2\gamma_-}{(1 + \rho\beta_-)^2 + \gamma_-^2} \tag{7}$$

An equation for the locus of all K with fixed α_- and $1 + \rho\beta_-$ and varying γ_- can be obtained by the elimination of γ_- from the equations for X and Y. Thus,

$$\left[X - 1 + \frac{\alpha_-^2}{2(1 + \rho\beta_-)}\right]^2 + Y^2 = \frac{\alpha_-^4}{4(1 + \rho\beta_-)^2} \tag{8}$$

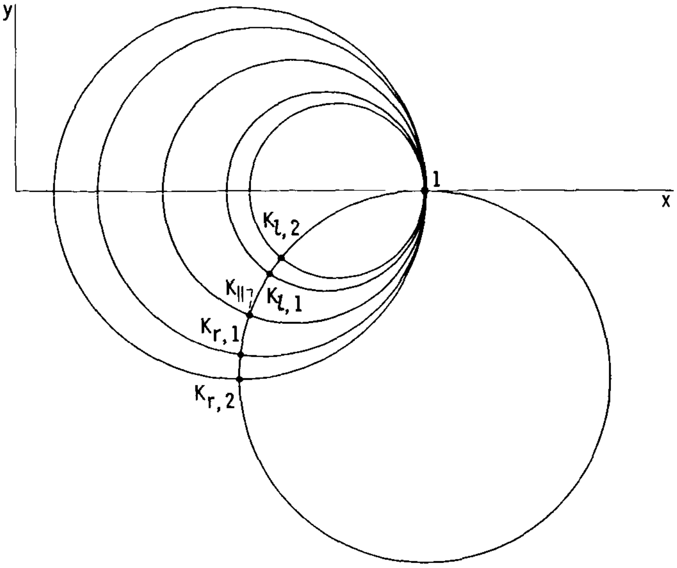

Figure 3. - Effect of changing cyclotron frequency; coefficients $K_{r,2}$ and $K_{l,2}$ correspond to lower cyclotron frequency.

These loci are circles which are centered at $X = 1 - \frac{\alpha_-^2}{2(1 + \rho\beta_-)}$ and which have radii of magnitude $r = \frac{\alpha_-^2}{2(1 + \rho\beta_-)}$. Similarly, equations for fixed α_- and γ_- and varying $1 + \rho\beta_-$ may be obtained. Thus,

$$(X - 1)^2 + \left(Y + \frac{\alpha_-^2}{2\gamma_-}\right)^2 = \frac{\alpha_-^4}{4\gamma_-^2} \tag{9}$$

All three values of ρ lead to this equation, which is a circle of radius $\alpha_-^2/2\gamma_-$, centered at $X = 1$, $Y = -\alpha_-^2/2\gamma_-$.

GEOMETRIC REPRESENTATION

The loci of $K_{||}$, K_ℓ, and K_r on the complex plane lie at the intersection of equation (9) and the appropriate equation (8). Figure 1 is a geometrical representation of these algebraic relations.

Cyclotron resonance occurs when $\beta_-^2 = 1$. In this case either the circle $\rho = 1$ or the circle $\rho = -1$ reduces to a vertical line through $X = 1$. It is thus appropriate to designate the lowest point on the circle $(1, -\alpha_-^2/\gamma_-)$ as $K_{resonance}$.

If the collision frequency is zero, the circle centered at $(1, -\alpha_-^2/2\gamma_-)$ reduces to the horizontal x-axis, and the dielectric coefficients are the intersections of the three circles with the x-axis: $K_{r,0}$, $K_{||,0}$, and $K_{\ell,0}$.

The effect of changing the collision frequency or the cyclotron frequency (and hence $\vec{B}$) is easily seen from the figures. An increase in the collision frequency γ_- reduces the radius of the circle, and the coefficients change as indicated in figure 2. The effect of changing the cyclotron frequency is indicated in figure 3.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 18, 1964.

APPENDIX - SYMBOLS

The notation is the same as that in reference 1.

B	magnetic field
$K_{\parallel}, K_{\ell}, K_r$	coefficients of diagonalized dielectric tensor
X	real part of K
Y	imaginary part of K
α_-	reduced plasma frequency, ω_{p_-}/ω
β_-	reduced cyclotron frequency, ω_{b_-}/ω
γ_-	reduced collision frequency, ν/ω
ν	collision frequency
ρ	parameter that identifies components of K
ω	wave frequency
ω_{b_-}	cyclotron frequency
ω_{p_-}	plasma frequency

REFERENCES

1. Allis, W. P., Buchsbaum, S. J., and Bers, A.: Waves in Anisotropic Plasma. M.I.T. Press, 1963, p. 21.
2. Deschamps, G. A., and Weeks, W. L.: Use of the Smith Charts and Other Graphical Construction in the Magneto-Ionic Theory. TN-60-468, Air Force Cambridge Res. Center, Apr. 26, 1960.

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